Deriving efficient frontiers for effort allocation in the management of invasive species

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Submitted to the Australian Journal of Agricultural and Resource Economics
Revision 1 (13/04/2010)

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Abstract

Invasive species cause significant losses through their effect on agriculture, human health and the environment. Their importance has increased with time due to globalisation, as the spread of invasive species is facilitated by the increased movement of people, cargo and genetic material around the world. There is a vast literature on the economics of invasive species and their management. Here we contribute to this literature by applying a spatio-temporal model to the allocation of surveillance resources. We focus on three questions regarding resource allocation to control a newly discovered invasion: the budget, which determines the amount of search effort available; the duration of the control program; and the allocation of surveillance and control in time and space. We also explore the complementary role of passive surveillance by members of the public. We derive efficient frontiers for effort allocation that represent the tradeoff between cost and probability of eradication after inefficient strategies have been eliminated. We use the results to illustrate how to evaluate whether introduction of passive surveillance is desirable based on cost and eradication probability. We conclude by discussing the implications of our findings in the design of control programs.

Keywords: invasive species, dispersal, spatio-temporal models, passive surveillance, search allocation,
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Introduction
Invasive species are one of the leading causes of global ecological change (D’Antonio and Vitousek 1992; Wilcove et al. 1998; Olson 2006), causing significant losses through their effect on agriculture, human health and the environment (Williams and Timmins 2002; Sinden et al. 2005; Colautti et al. 2006). The spread of invasive species is facilitated by the increased movement of people, cargo and genetic material around the world and there is an expectation that climate change will further increase spread, by changing suitable habitat and causing adverse weather events that support the spread of disease vectors (Mooney and Hobbs, 2000; McNeely et al. 2001).

When new invasions are discovered the first decision to be made is whether to attempt eradication or containment (Cacho et al. 2008). For weeds, where a persistent seed bank normally develops, eradication is usually a reasonable goal if three conditions are met: the infestation is small (Rejmanek and Pitcairn 2002), the infestation is delimited properly (Panetta and Lawes 2005) and there are sufficient resources. Successful eradication of pest animals presents similar challenges (Bomford et al. 1995); in general the probability of eradication will be highest, and eradication achievable at lowest cost, when an invasion is detected at a stage when the species is neither abundant nor widely distributed (Panetta 2009).

Resources for biosecurity and response to invasions are invariably limited, and eradication programs can be expensive so it is essential that cost-effective strategies be chosen. Many aspects of the resource-allocation problem have been addressed in the literature. These include consideration of the optimal level of search effort required to detect the invader given its biological characteristics and the damages that might result from its spread (Cacho et al. 2007; Mehta et al. 2007), as well as the cost implications of decisions on effort allocation to remove low density populations at the edge of an invasion or older, core populations which
are easier to find (Taylor and Hastings 2004), or to target new foci ahead of the main invasion (Moody and Mack 1988).

Biological invasions are complex dynamic systems with many sources of uncertainty and generally exhibiting strong geographical variation. Therefore in many cases it is important to consider how surveillance resources should be allocated across a landscape where habitat suitability and searcher effectiveness vary (Cacho et al. 2009; Hauser and McCarthy, 2009). As an option to avoid spending large amounts of money searching for invaded sites over a large area, pest management agencies use ‘passive surveillance’: reports from members of the public of encounters with pests, to assist in surveillance and control (MAFBNZ 2008, Beale et al. 2008). Cacho et al. (2009) showed that the probability of eradication can increase and total costs of managing an invasion can be reduced considerably if passive surveillance is introduced, provided enough resources are available to apply the necessary search and control effort in response to all detections by the public. By enlisting the help of the public to detect infested sites, the agency is able to achieve broader-area coverage and allocate its existing search resources more efficiently.

In this paper we concentrate on the allocation of search effort based on three decisions for a newly discovered invasion: the budget, which determines the amount of search effort available; the duration of the control program; and the allocation of surveillance and control in time and space. We derive efficient frontiers for effort allocation that represent the tradeoff between cost and probability of eradication when search resources are allocated efficiently. So far as we know this is the first time the concept of an efficient frontier is applied to invasive species in this way. We start by describing the model and the decisions that the agency in charge of controlling an invasion must make. Next we present the method, which consists of a series of computer experiments using a spatio-temporal model of spread. This is followed by presentation of results and derivation of efficient frontiers. We conclude by discussing the
implication of our findings for the design of invasive-species control programs and identifying future research needs.

**The Model**

Consider an agency in charge of eradicating an invasion that has been recently detected. The agency has three decisions to make: the size of the budget; the expected program duration; and the allocation of search and control effort in time and space. We assume that the agency has already made the preliminary decision on whether eradication should be the objective, rather than containment or doing nothing. The agency is also considering whether to use passive surveillance within its portfolio of search and control strategies. If passive surveillance is adopted the public will be offered a bounty payment ($C_B$) for each detection reported to the agency. The budget for the program must be able to cover expected costs. The cost of the operation ($C$) depends on the amount of search undertaken by the agency, the number of reports by the public and the cost of treatment:

$$C = \sum_{t} \left( N_{pt} C_B + N_{At} m C_m + A_{Tt} C_T \right) + \sum_{t=1}^{t-S_R} \left( N_{At} m C_m \right) (1 + \beta)^{t}$$

(1)

where $N_{pt}$ is the number passive finds reported in year $t$, $N_{at}$ is the area searched (ha), $m$ is the annual search effort (h/ha), $A_{Tt}$ is the number of cells treated, $\beta$ is the discount rate, and $C_B$, $C_m$ and $C_T$ are the bounty payment ($$/report), the cost of searching ($$/h) and the cost of treatment ($$/ha), respectively. The second summation term in (1) represents the cost of repeat searches that are undertaken as a result of detections in the previous $S_R$ years. The variables $N_{pt}$, $N_{at}$ and $A_{Tt}$ are calculated through stochastic simulations with a spatially-explicit model containing a heterogenous habitat (Cacho et al. 2009).

The landscape is represented by a rectangular square lattice of dimensions $n_r$ (rows) by $n_c$ (columns) containing $n=n_r \times n_c$ cells of one hectare each. For simplicity, variables associated
with the landscape are identified by index $i=1,...,n$; numbered sequentially down the rows and then across columns rather than by their Cartesian coordinates. Cells have four attributes: habitat suitability ($\alpha_i$), with possible values $0 \leq \alpha_i \leq 1$; detectability ($\lambda_i$), measured in metres; search speed ($s_i$), in m/h; and ownership type ($o_i$), with possible values of 0 (public) and 1 (private). The state of a cell is given by its invasion status $x_i$ (1=present, 0=absent). For convenience all variable definitions are presented in Table 1. Detectability ($\lambda$) represents effective sweep width, a simple measure used in search theory. This parameter is derived by converting a bell-shaped lateral range curve (showing the probability that the target will be detected as a function of its distance from the searcher) into a rectangle of height 1, $\lambda$ is the width of this rectangle. For more details see Cacho et al. (2007).

[Table 1 here]

**Dispersal**

At the start of a simulation an invasion is introduced in a random location. An invaded cell produces $w$ propagules per time period, and these propagules spread to neighbouring cells. The distance between cells $(d_{ij})$ determines the proportion of propagules from cell $i$ that reach cell $j$ according to a dispersal kernel. A Cauchy kernel was assumed with dispersal parameter $\gamma$ (Kot et al. 1996, Cacho et al. 2009).

The dispersal process is driven by an adjacency matrix $A$ of dimensions $n \times n$, whose element $A_{ij}$ represents the probability that a propagule originating in cell $i$ will land on cell $j$. The probabilities in each row $A_i$ are derived from the dispersal kernel, and $\sum_j A_{ij} = 1$, so that each propagule is guaranteed to land somewhere within the landscape. This implicitly assumes the landscape is large enough that there are no escapes beyond its boundaries (or alternatively it represents an island). The probability that a propagule will survive and establish a new
infestation depends on the habitat suitability of the cell where it lands. The probability that a
given site will be invaded is given by an invasion probability vector \((\mathbf{p})\):

\[
\mathbf{p} = 1 - \exp(-\mathbf{a} \cdot \mathbf{y}) \tag{2}
\]

\[
\mathbf{y} = (\mathbf{x}, w) \mathbf{A} \tag{3}
\]

The vector \(\mathbf{p}\) represents a probability map that incorporates the combined effect of invaded
sites \((\mathbf{x})\) and habitat suitability \((\mathbf{a})\). To implement stochastic dispersal, \(\mathbf{p}\) is compared to a
vector of uniform random numbers \(\mathbf{r}\) and the new state of each cell is set according to the
rule:

\[
x_i = 1, \text{ if } r_i \leq p_i \tag{4}
\]

otherwise \(x_i\) remains in its current state. Long-distance dispersal can occur with probability \(p_L\)
independently of the dispersal kernel, as may occur when propagules are transported by road,
water, or other means. A long distance jump from an invaded cell occurs if:

\[
 r_i \leq p_L x_i \tag{5}
\]

where \(r_i\) is a random number drawn form a uniform distribution. The destination of these
jumps is selected randomly within \(\mathbf{x}\).

**Surveillance and Control**

An invaded cell can be detected through passive surveillance with probability \(q_i\) or through
active surveillance with probability \(z_i\). In passive surveillance the public detects an invader
and reports it to the agency. The probability of passive detection depends on the ownership
attributes of cells; for any cell \(i\):

\[
q_i = p_p (o_i) + p_u (1 - o_i) \tag{6}
\]

where \(p_p\) and \(p_u\) are passive detection probabilities in private, and public, land, respectively.
The probability of detection through active surveillance is calculated based on search theory
(Cacho et al., 2006, 2007). The probability that an invasion in cell \(i\) will be detected depends
on the search effort applied $m_i$ (h/cell), the speed of search $s_i$ (m/h), effective sweep width $\lambda_i$ (m) and the area of the cell $a$ (m$^2$):

$$z_i = 1 - \exp\left( -\left( \frac{s_i m_i \lambda_i}{a} \right) \right)$$  \hspace{1cm} (7)

The expression within the inner brackets in (7) measures coverage: the numerator is the area effectively searched (m$^2$) as the product of distance traversed ($s_i \times m_i$) times effective sweep width ($\lambda_i$); the denominator is the area of the cell (m$^2$). Effective sweep width ($\lambda_i$) measures the detectability of the target, its derivation is explained by Cacho et al. (2007).

Only cells where invasions have been detected are treated, and invaders are killed with probability $p_k$. The probability that an infestation will be eliminated in cell $i$ is given by the probability of finding the infestation ($z_i$) times the probability that treatment will be successful:

$$k_i = x_i z_i p_k$$  \hspace{1cm} (8)

This probability is controlled by the agency through search effort ($m_i$ in equation 7). Other parameters in the search function ($s_i$, $\lambda_i$ and $a$) are given by the environment and by the features of the invasive organism, and they are kept constant throughout a simulation.

**Method**

Starting with state $x_0 = [0]$ at $t=0$, a stochastic simulation is seeded by setting cell $x_i=1$, where $i$ is randomly selected, and then applying equations (2) to (5) iteratively until $t$ equals the minimum discovery time ($t_D$). Once $t=t_D$ the possibility of discovering the invasion is switched on by applying equations (6) and (7) against random numbers. When the invasion is discovered within a simulation, $t$ is set back to zero and the control program commences.
Simulation of the control program is executed by first applying equations (6) to (8) to represent the process of finding and treating infestations and then updating the state, from 1 to 0, based on the probability of kill (8) of each cell $i$ that meets the condition:

$$ r_i \leq k_i x_i $$

(9)

where $r_i$ is a random number drawn from a uniform distribution. Dispersal of the remaining population is then represented by applying equations (2) to (5) to the updated state vector. This process is applied iteratively for a planning horizon of $T$ periods.

We assume that, each year, search effort is invested in the following activities: (i) searching sites where treatment has occurred in the recent past (repeat search); (ii) in response to reports from the public (follow-up search); and (iii) through independent surveillance in public land not previously searched during (i) or (ii) (active search). The search process followed is a form of adaptive cluster sampling (Philippi, 2005, Smith et al., 2003, Thompson, 1990). In this method a radius ($r_m$) is searched around each detection and additional radii are searched around any additional detections resulting from this search, after eliminating overlap. The process continues until no more detections are made.

We apply the soft budget constraint:

$$ N_{At} m + \sum_{t=1}^{t-1} N_{At} m \leq M n $$

(10)

where $N_{At}$ is the total number of cells searched, from equation (1), $m$ is search effort per cell, $M$ is total annual effort available and $n$ is the total number of one-hectare cells. This constraint can be overridden by the need to follow up on all passive detections. This essentially means that emergency funds are available to deal with excessive passive detections, but any additional active search is treated as a discretionary expense limited by the budget.
Our map had dimensions $n_r=n_c=128$, therefore the total number of cells ($n$) is 16,384, which is the same as the total area because each cell is one hectare in size. Therefore $M=1$ represents 16,384 hours of search available to be distributed spatially in any pattern that may arise as a result of probabilistic detections combined with the control parameters.

Artificial maps of habitat suitability and ownership type were generated using a fractal algorithm (Saupe, 1988). These maps were the same as those reported in Figures 3A and 3C of Cacho et al. (2009). The model was implemented in Matlab (The Mathworks, 2002).

The parameter values used in the simulations reported below are presented in Table 2. The simulations do not represent any specific species, but apply in general to sessile invaders (those whose location remains fixed after establishment), which include plants, insects that build nests such as wasps and ants, and the adult stages of many aquatic organisms.

[Table 2 here]

The total area invaded at time $t$ for a single iteration of the model is represented by:

$$X_t = \sum_i x_{it}$$

(11)

$X_t$ is used to calculate the probabilities of containment and eradication. Eradication is defined as absence of invaded sites at time $t$, containment is defined as a situation where $X_t \leq X_0$.

A series of experiments were performed, whereby $N$ Monte Carlo iterations were executed, each with a planning horizon ($T$) of 10 years. We assume that four control parameters are considered by the agency:

1) The total search effort available per time period ($M$), expressed as a proportion of the total area that could be searched; five values were tested ($M = 0.2, 0.4, 0.6, 0.8$, ...
1.0). This variable is directly related to the budget and drives the soft constraint (10).

2) The amount of effort applied per cell \((m)\), expressed as hours per ha; five values were tested \((m = 2, 4, 6, 8, 10)\).

3) The radius searched around detections \((r_m)\), expressed in number of cells; three values were tested \((r_m = 5, 10, 15)\).

4) The number of repeat searches \((S_R)\); four value were tested \((S_R = 0, 1, 2, 3)\).

The values tested for each parameter were combined into a full factorial design with \(5 \times 5 \times 3 \times 4\) levels = 300 experiments. The maximum value of \(M\) tested was 1.0, which represents enough effort to cover every hectare on the map with one hour of search \((16,384\) hours given the size of the map). Increases beyond \(M=1\) mean that more than one hour is available to search each hectare; but does not mean that all cells on the map will be searched. The actual distribution of effort in space varies depending on the search strategy used and on the detections that occur during a simulation.

For each experiment we calculated the total cost of the program using equation (1) and the probability of eradication \((P_E)\) and probability of containment \((P_C)\) as:

\[
P_{Ei} = \frac{\sum_j \left(X_{ij} = 0\right)}{N}\quad (12)
\]

\[
P_{Ci} = \frac{\sum_j \left(X_{ij} \leq X_{j0}\right)}{N}\quad (13)
\]

where \(j=1,\ldots,N\) denotes a Monte Carlo iteration and \(N=500\) in this application.
**Results and Discussion**

The first step in the analysis is to understand how the budget available to the agency affects the probability of success. This can be estimated for a particular search strategy by varying the amount of search effort available to the agency \( (M) \) and calculating the probability of success and the associated costs. Success is defined in terms of achieving containment and/or eradication within a given time horizon (equations 12 and 13) and cost is defined in equation (1). A selection of results where \( M \) varies and other search parameters are kept constant (Figure 1) reveals that, for the given search strategy (in terms of \( m, r_m \) and \( S_R \)), the probability of success increases markedly as \( M \) increases from low values and flattens out as \( M \) approaches 0.8 of the area at risk (Figure 1A). As the amount of search effort available increases the total cost tends to decrease (Figure 1B), because the invasion is more likely to be eradicated earlier. These curves will shift as other model parameters change but similar patterns will arise. Of particular interest is the effect of surveillance strategies for a given budget, which are defined in terms of search parameters \( (m, r_m \) and \( S_R \)) and the probabilities of passive detection in private and public areas \( (p_p \) and \( p_u \)).

![Figure 1 here]

**Dominance and efficient sets**

We now focus on two measures of performance identified above: probability of eradication \( (P_E) \) for a given program duration (equation 12) and total cost of the program \( (C) \) measured in present-value terms (equation 1). A strategy \( i \) is said to dominate strategy \( j \) if the following conditions are satisfied:

\[
PE(i) \geq PE(j) \\
C(i) \leq C(j)
\]

(14)

and the strict inequality applies for at least one of these two conditions.
The concept of dominance allows us to compare search strategies and eliminate those that are inefficient. Figure 2 presents the 300 strategies tested plotted in $P_E$-$C$ space. The plots illustrate four different program durations (Figure 2, panels A to D). Each point in these plots represents the mean of 500 iterations of the model with random dispersal and probabilistic search as explained earlier.

In these plots (Figure 2) the ideal position is on the top left corner, where $P_E = 1.0$ and $C = 0$. This position dominates all strategies that have positive costs and could represent the do-nothing option for an invasion that is not viable and will die out by itself, which obviously is of no interest to the agency represented in this study. Dominated strategies can be identified graphically as those points that are below and to the right of at least one other strategy. Dominated strategies can be eliminated from further analysis, and the remaining non-dominated strategies form the efficient set for the given experimental design.

The efficient set in Figure 2 moves up and to the left as program duration increases. For a four-year program the highest probability of eradication is 0.56 with an expected cost of $1.64M$ (Figure 2A). Whereas in a 10-year program the highest probability of eradication is 0.88 with an expected cost of $2.18M$ (Figure 2D). If a $P_E$ value of 0.88 is not satisfactory to the agency it may be possible to increase the budget and identify other efficient strategies for the higher search effort available. But it would be inefficient to increase search effort indefinitely, as there are diminishing marginal returns to search effort in terms of eradication probability (see Figure 1A). A better alternative may be to introduce passive surveillance.

*Introducing passive surveillance*

Figure 3 presents the same set of 300 strategies as those of Figure 2, but in this case the probabilities of passive detection in private and public areas ($p_p$, $p_u$) are set at (0.5, 0.1)
instead of (0,0). The effect of introducing passive surveillance is dramatic: the eradication probability increases substantially and the cost decreases compared to cases with no passive surveillance (compare Figure 3 and Figure 2). A relatively uniform pattern arises when passive surveillance is introduced. For a given value of $M$, changing other search parameters ($m$, $r_m$ and $S\beta$) causes points to shift in a SE to NW direction forming an elongated arrangement. Each value of $M$ has at least one efficient point; as $M$ increases the efficient points move in a NE direction, showing increases in $P_E$ at higher $C$. This is particularly evident in Figure 3A, which also shows that when passive surveillance is introduced into a four-year program the highest probability of eradication is 0.91 and the expected cost is $0.93M.

[Figure 3 here]

With enough resources and a 10-year program, success is virtually assured (Figure 3D), with a maximum eradication probability > 0.99. Although introducing passive surveillance results in higher eradication probabilities at lower costs, these results do not mean that strategies which include passive surveillance (Figure 3) dominate strategies that do not (Figure 2). This is because the cost of achieving the prescribed passive detection probabilities (0.5, 0.1) was not included in the analysis.

**Efficient frontiers**

Suppose the agency in charge of controlling the invasion wishes to achieve the highest possible eradication probability within 4 years and is considering introducing passive surveillance. The two relevant strategies to determine whether this move is worth taking are the highest points in Figures 2A and 3A. The $(P_E, C)$ pairs for these two points are (0.56, 1.64) and (0.91, 0.93), representing a 0.35 increase in eradication probability and a $710,000 reduction in cost as a result of introducing passive surveillance. The relevant question now becomes: can we achieve the desired increase in passive detection probabilities from (0, 0) to
(0.5, 0.1) at a cost of $710,000? If the answer is positive we have a Pareto improvement when passive surveillance is introduced, because the same budget of $1.64M achieves a higher eradication probability (0.91 compared to 0.56). This illustration is for a four-year program, but a different program duration may be more efficient.

Overall efficient frontiers can be derived from Figures 2 and 3, for the cases without or with passive surveillance respectively, by pooling all the points in plots A to D and eliminating those that are dominated. The two frontiers in Figure 4 were derived through this process. We cannot unambiguously declare that points on the left frontier (with passive surveillance, represented by circles) dominate any of the points on the right frontier (without passive surveillance, represented by triangles), because the cost of achieving the frontier shift is not included in the analysis. This cost is unknown and highly uncertain, given the unpredictability of the public’s response to awareness campaigns.

[Figure 4 here]

Details of an arbitrary subset of points from Figure 4 are presented in Table 3. Point $e$, for example, represents the case where no passive surveillance occurs and: the total search effort available is 16,384 h ($M=1.0$), the effort applied to each parcel searched is 10 h/ha ($m=10$), the search radius around detections is 1 km ($r_m=10$ cells), and 2 repeat searches are applied to sites that have been previously treated ($S_R=2$). This strategy will achieve an eradication probability of 0.88 at a cost of $2.18M and is expected to last a maximum of 10 years (Table 3) but may be completed earlier.

[Table 3 here]
Suppose the agency is considering shifting to the left frontier in Figure 4 by introducing passive surveillance, and that the intention is to move from point $e$ to point $C$. This would cause expected cost to decrease by $1.46M (from $2.183M to $0.727M) and eradication probability to increase from 0.88 to 0.99 (Table 3). Achieving passive surveillance at the level required for this shift, where $(p_p, p_u) = (0.5, 0.1)$, will involve a number of costly public-awareness activities. If the cost of these activities adds to $1.46M or less in present value terms, then the decision to move from $e$ to $C$ involves a Pareto improvement and should be adopted, because the probability of eradication increases from 0.88 (point $e$) to 0.99 (point $C$) at no additional cost (Table 3).

In this example, $1.46M should be seen as the minimum amount the agency should be prepared to spend to increase passive surveillance. The optimal expenditure may be higher than this if the benefits of control are considered. The shift from point $e$ to point $C$ not only increases eradication probability within a 10-year program, but it also increases the probability of early eradication (Figure 5), which may bring benefits that have not been measured here.

[Figure 5 here]

As an example of why passive surveillance may be worth more than estimated above, note in Figure 5 that the probability that the invasion will be eradicated within 2 years is 0.6 under strategy $C$, whereas under strategy $e$ the equivalent probability is 0.32. This suggests that even if the shift from $e$ to $C$ costs more than $1.46M to achieve, the move may still be considered a good investment when the benefits of early eradication are considered. These benefits come from avoided damages associated with the difference between curves $C$ and $e$ in Figure 5. The monetary value of this area ($C-e$) can be calculated if two additional pieces of information are available: a function linking pest presence to flows of environmental services and other economic impacts, and a function describing the demand for the environmental services.
affected by the invasion. The former requires additional biophysical modelling to represent
the physical damages of the invasion, whereas the latter can be estimated through choice-
modelling or contingent-valuation surveys (Amigues et al., 2002; Loomis and White, 1996;
Van Bueren and Bennett, 2004). An agency considering whether to attempt eradication may
decide to devote part of their budget to valuation of damage in order to take a more informed
decision, but this will take some time and may not be practical in the initial stages when rapid
response is required. The absence of a damage measure in our model, however, does not
invalidate its usefulness as a tool for conducting cost-effectiveness analysis. Furthermore,
extension of the model to capture damage is straightforward. If the functions required are
available, damage can be added as an additional cost in equation (1).

Implications of findings and further research needs

The problem of allocating resources in space and time is not easy. Ideally, we would apply
optimal control techniques that provide a state-based decision approach. A Markovian
technique such as stochastic dynamic programming (SDP) is often the method of choice to
derive decision rules based on the state of the system at any time in a stochastic environment
(i.e. Shea and Possingham 2000). However, SDP is not well suited to spatially-explicit
problems such as ours because the state space is so large. Defining the state as presence /
absence for each cell in the landscape would result in $2^{16384}$ possible invasion states in our
example, and the set of Markov transition probability matrices for this problem will contain
the square of this number of elements (many of them zeros) for each possible decision. An
option to overcome this dimensionality problem is to derive simplified state variables that
capture the essence of the problem and then populate the probability matrices using Monte
Carlo simulation for a limited set of possible states. The simplified states may be defined in
terms of state variables such as total area invaded, average population density and degree of
clustering of individual infestations. Bogich and Shea (2008) and Hyder et al. (2008) are
examples of studies that use simplified states in invasive species problems. These decision approaches are elegant and useful, but they may result in loss of information relevant to the decision process, and they can handle only one objective function at a time.

Our approach does not require simplification of the state space and allows us to consider the trade-off between the budget and the probability of success in invasive control programs. We identify efficient frontiers by running Monte Carlo simulations for a given experimental design. While our method has the drawback that it is not state based, it allows us to identify inefficient strategies and eliminate them from further consideration. The efficient points that remain represent the minimum cost for a given probability of success, or the maximum probability of success for a given cost, within the set of alternatives contained in the experimental design.

Our findings indicate that, as the annual budget increases, individual parcels should be sampled more intensively first, then the duration of the program should be increased, and finally repeat treatments should be used. As these changes occur, both the probability of eradication and the total cost of the program increase (see Table 2). Results also indicate that introducing passive surveillance can reduce total cost (Figure 4) and increase the probability of early eradication (Figure 5). But introducing passive surveillance involves costs for activities that were not considered here. These activities may include information campaigns in the media; establishment of a hotline to receive public reports; educational visits to schools, clubs and other community organisations; and perhaps payment of rewards for true positive reports. Additional expenses may be caused by a large number of false positives that require crews to travel to the site and search areas that are not infested. Of these additional expenses, only the cost of reward payments was considered in our analysis. Although the costs of enabling passive surveillance are unknown, our method allows us to estimate the minimum amount that should be spent on this, based on the size of the leftward shift in the frontiers in
Figure 4. An agency can use this estimate as guidance to decide whether to invest in the establishment of an information campaign that may involve substantial upfront fixed costs.

Although efficient frontiers are common in economics, to our knowledge this is the first time that the concept is applied to the tradeoff between cost and probability of success in invasive control programs. The efficient frontier was derived from an arbitrary experimental design (a complete factorial in our case) and does not represent a global optimum unless the decision space is sampled exhaustively, which is not practical with large spatial models. The method greatly simplifies the process of eliminating a large number of poor management strategies and helps select a small set of ‘good’ strategies. Furthermore, the method provides a clear relationship between the budget and the probability of success, which can be very useful to pest-control agencies and policy makers.

As a possible extension of this work, an alternative to using arbitrary experimental designs is to use a genetic algorithm to drive the process of identifying efficient strategies (i.e. Cacho and Simmons 1999; Hester and Cacho 2009). The genetic algorithm would set the effort allocations through an evolutionary process, rather than relying on a fixed experimental design. It should be possible to design an algorithm that makes use of the dispersal probability map generated by the model (see equation 2) and that considers what is known about the state of the invasion at any time to make more informed decisions. This application would require careful thought regarding the representation of the learning process (through evolution of a population of control parameters) as the invasion progresses.

In further applications of our method, there are important policy decisions which could be studied by manipulating model parameters. For example, the proportion of treated organisms that are killed \( (p_k) \) can be used to study the benefits of investing in technology that improves treatment effectiveness; and the detectability parameter \( (\lambda) \) can be used to evaluate improvements in detection technology, such as the use of dogs or electronic sensors to detect
invaders. The desirability of these investments on new technologies can then be assessed based on their effects on eradication probabilities and costs. Many other policy and allocation questions can be studied following a similar process.

**Acknowledgments**

This research was undertaken with funding from the Australian Centre for Excellence in Risk Analysis under ACERA project 0806.

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Table 1. Variable definitions and the equation numbers where they are first used.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>Cost of control program (present value)</td>
<td>1</td>
</tr>
<tr>
<td>$N_{pt}$</td>
<td>Number of passive finds at time $t$</td>
<td>1</td>
</tr>
<tr>
<td>$N_{at}$</td>
<td>Number of active finds at time $t$</td>
<td>1</td>
</tr>
<tr>
<td>$A_{Ti}$</td>
<td>Number of passive finds at time $t$ = area treated (ha)</td>
<td>1</td>
</tr>
<tr>
<td>$p$</td>
<td>Invasion probability vector with elements $p_i$</td>
<td>2</td>
</tr>
<tr>
<td>$x_t$</td>
<td>State vector (presence/absence) at time $t$, with elements $x_{it}$</td>
<td>3</td>
</tr>
<tr>
<td>$A$</td>
<td>Adjacency matrix with elements $a_{ij}$</td>
<td>3</td>
</tr>
<tr>
<td>$y$</td>
<td>vector of number of propagules landing in each cell</td>
<td>3</td>
</tr>
<tr>
<td>$q_i$</td>
<td>probability of passive detection in cell $i$</td>
<td>6</td>
</tr>
<tr>
<td>$o$</td>
<td>vector of ownership attributes (urban/rural) with elements $o_i$</td>
<td>6</td>
</tr>
<tr>
<td>$z_i$</td>
<td>probability of active detection in cell $i$</td>
<td>7</td>
</tr>
<tr>
<td>$k_i$</td>
<td>probability that infestation in cell $i$ will be eliminated</td>
<td>8</td>
</tr>
<tr>
<td>$X_t$</td>
<td>Total area invaded at time $t$</td>
<td>11</td>
</tr>
<tr>
<td>$P_{Et}$</td>
<td>Probability of eradication up to time $t$</td>
<td>12</td>
</tr>
<tr>
<td>$P_{Ct}$</td>
<td>Probability of containment up to time $t$</td>
<td>13</td>
</tr>
</tbody>
</table>
Table 2. Parameter values used in simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Environmental and biological assumptions:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td>100</td>
<td>propagule pressure</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.02</td>
<td>habitat suitability (mean)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>5</td>
<td>effective sweep width (m)</td>
</tr>
<tr>
<td>$p_k$</td>
<td>0.98</td>
<td>treatment effectiveness</td>
</tr>
<tr>
<td>$p_L$</td>
<td>0.01</td>
<td>probability of long-distance jump</td>
</tr>
<tr>
<td>$t_D$</td>
<td>3</td>
<td>minimum time to discovery (y)</td>
</tr>
<tr>
<td>$s$</td>
<td>1000</td>
<td>search speed (m h$^{-1}$)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3.95</td>
<td>dispersal kernel parameter</td>
</tr>
<tr>
<td>$D$</td>
<td>10</td>
<td>maximum dispersal distance (number of cells)</td>
</tr>
<tr>
<td><strong>Invasion management assumptions:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_p$</td>
<td>0, 0.5</td>
<td>probability of passive detection, (private)</td>
</tr>
<tr>
<td>$p_u$</td>
<td>0, 0.1</td>
<td>probability of passive detection, (public)</td>
</tr>
<tr>
<td>$M$</td>
<td>*</td>
<td>total effort available per year (proportion of total area)</td>
</tr>
<tr>
<td>$m$</td>
<td>*</td>
<td>search effort per cell (h/ha)</td>
</tr>
<tr>
<td>$r_m$</td>
<td>*</td>
<td>search radius for detected sites (no. of cells)</td>
</tr>
<tr>
<td>$S_R$</td>
<td>*</td>
<td>number of repeat searches</td>
</tr>
<tr>
<td><strong>Economic assumptions:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_B$</td>
<td>500</td>
<td>cost of bounty ($ per find)</td>
</tr>
<tr>
<td>$C_m$</td>
<td>30</td>
<td>cost of search ($ h^{-1}$)</td>
</tr>
<tr>
<td>$C_T$</td>
<td>100</td>
<td>cost of treatment ($ ha^{-1}$)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.06</td>
<td>discount rate</td>
</tr>
<tr>
<td>$a$</td>
<td>10,000</td>
<td>cell area ($m^2$)</td>
</tr>
<tr>
<td>$T$</td>
<td>15</td>
<td>planning horizon (y)</td>
</tr>
</tbody>
</table>

* These parameters values are varied in the analysis as explained in the Method section.
Table 3. Selected results for points on the frontier, point labels correspond to those in Figure 4.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Results</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Point</td>
<td>$M$</td>
<td>$m$</td>
</tr>
<tr>
<td>$a$</td>
<td>0.2</td>
<td>6</td>
</tr>
<tr>
<td>$b$</td>
<td>0.4</td>
<td>6</td>
</tr>
<tr>
<td>$c$</td>
<td>0.6</td>
<td>8</td>
</tr>
<tr>
<td>$d$</td>
<td>0.8</td>
<td>8</td>
</tr>
<tr>
<td>$e$</td>
<td>1.0</td>
<td>10</td>
</tr>
</tbody>
</table>

with no passive surveillance

<table>
<thead>
<tr>
<th>Point</th>
<th>$M$</th>
<th>$m$</th>
<th>$r_m$</th>
<th>$S_R$</th>
<th>Cost ($SM$)</th>
<th>$P_E$</th>
<th>Duration (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>0.2</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0.366</td>
<td>0.412</td>
<td>4</td>
</tr>
<tr>
<td>$B$</td>
<td>0.4</td>
<td>8</td>
<td>10</td>
<td>2</td>
<td>0.515</td>
<td>0.630</td>
<td>4</td>
</tr>
<tr>
<td>$C$</td>
<td>0.6</td>
<td>8</td>
<td>10</td>
<td>2</td>
<td>0.727</td>
<td>0.988</td>
<td>10</td>
</tr>
<tr>
<td>$D$</td>
<td>0.8</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0.865</td>
<td>0.999</td>
<td>10</td>
</tr>
</tbody>
</table>

with passive surveillance
Figure 1. Effect of search effort available on (A) probabilities of eradication and containment within 10 years (given by equations 12 and 13) and (B) the corresponding cost in present-value terms (given by equation 1).
Figure 2. Results of 300 experiments plotted in eradication probability-cost space; each point represents the mean of 500 iterations with a given search strategy, passive detection probability = (0,0).
Figure 3. Results of experiments plotted in eradication probability-cost space; each point represents the mean of 500 iterations with a given search strategy, passive detection probability = (0.5,0.1).
Figure 4. Dotted lines represent approximations to the efficient frontiers with passive detection probability of zero (triangles) or 0.5 in private parcels and 0.1 in public parcels (circles); detailed results for the arbitrary set of points labelled $a$ to $e$ and $A$ to $D$ are presented in Table 3.
Figure 5. Eradication probability against time for selected points on the efficient frontiers with no passive surveillance (e) and with passive surveillance (C); the labels relate to the points presented in Figure 4 and Table 3.