Risk and the Limitations of (Classical) Probability Theory

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Outline

High Stakes and Low Probability

- Risk analysis is typically concerned with low probability and low utility events.
- I'll mostly be focussing on issues concerning the treatment of small probabilities
- For many risk analysis purposes we would like a more discriminating treatment of low probabilities.

The Problem of Measure-Zero Events

- Classical probability theory takes unconditional probability theory as primitive and defines conditional probability as a ratio of unconditional probabilities:

$$Pr(A|B) = Pr(A \cap B)/Pr(B), \quad \text{where} \;\; Pr(B) \neq 0$$

- But sometimes this gives strange results.
- Consider the probability of an event given some measure-zero event.
- Standard probability theory has it that all such conditional probabilities are undefined.
- E.g. The probability of being in the Eastern hemisphere given that you're on the equator.

Impossible of Just Unlikely?

- Standard probability theory is not discriminating enough.
- Measure zero (but possible and in some cases *actual*) events are assigned probability zero.
- Impossible events are also assigned probability zero.
- Perhaps “undefined” is the right answer for the probability of an event conditional on an impossible event, but it is definitely *not* the right answer for other cases.

Why Worry?

- Many events we are interested in have probabilities so small that zero is the most natural number to assign as their probability.
- Sometimes the data suggests that probability zero is the right answer, even though the event is not impossible.
- Neglecting to recognise some possible states in a decision problem is in effect to assign that state probability zero (e.g. probability of a coin landing on its edge).
- Worse still, events of measure zero actually occur!
- The first three points concern the (perhaps erroneous) assignment of probability zero to some events, but the last point is about the existence of probability zero events.

Non-Standard Analysis

- One possible solution is to invoke non-standard analysis (and hence, non-standard probability theory).
- There are different versions of this but the basic idea is that there are infinitely many (non-standard) infinitesimals that are larger than zero but smaller than any of the positive reals.
- These infinitesimals can be employed for possible events and reserve zero for impossible events. (Though there is still the problem of which infinitesimal to use.)
- Another solution is to take conditional probabilities as primitive and define unconditional probabilities in terms of the conditional probabilities (Hájek, 2002).

Vagueness

- A predicate is vague iff it permits borderline cases (e.g. *acceptable* risk, *endangered* species).
- The usual laws of logic break down in the presence of vagueness. (E.g. it is not the case that a risk is either acceptable or not—the risk might be *borderline* acceptable).
- In risk analysis, and in decision theory more generally, events can be vague (e.g., the event that an unacceptable number of heat tiles on the space shuttle will detach).
- Actions can also be vague (e.g. the action of implementing a captive breeding program for an endangered species).

Uncertainty Due to Vagueness

- Vagueness can give rise to a particularly nasty kind of uncertainty (e.g. how many endangered species?).
- This uncertainty persists, even in situations of perfect information.
- Classical probability theory is based on classical logic and so is not suited to the task of dealing with uncertainty due to vagueness.
- There are, fortunately, various tools such as vague probabilities (e.g. Walley, Dempster-Shafer, info-gap).
- There are also various non-classical logics: multi-valued logics (three-valued logics, fuzzy logics), supervaluations, modal logics, and intuitionistic logic.
- The latter can be particularly useful in settings where qualitative answers are sufficient.

Supervaluational Logic

- The basic idea is that we consider all permissible sharpenings of the concept in question.
- If some statement is true according to all such sharpenings, then it is true; If a statement is false according to all such sharpenings, then it is false; if it is true on some sharpenings and false on others, it is neither true nor false.
- The resulting logic preserves classical theorems (e.g. excluded middle) and is seen by many as the minimal deviation from classical logic required to deal with vagueness.

Summary

- Low probability events and vague events can present problems for classical probability theory (especially when occurring together).
- There are various tools for dealing with these problems but these tools involve significant departures from classical (Kolmogorov) probability theory.

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