How to Evaluate and Manage Risk When We Don't Know Probabilities

Yakov Ben-Haim

Technion

Israel Institute of Technology

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1 *HIGHLIGHTS*

¶ **Policy evaluation and selection under severe uncertainty.**

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- **Why we can't guess probabilities (and shouldn't try).**

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under severe uncertainty. Main ideas:

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- **Info-gap models for severe uncertainties: in models, spatial distributions, pdfs.**

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under severe uncertainty. Main ideas:

- **Why we can't guess probabilities (and shouldn't try).**
- **Info-gap models for severe uncertainties: in models, spatial distributions, pdfs.**
- **Example: robustness-premium for sub-optimal policy.**

¶ Policy evaluation and selection

under severe uncertainty. Main ideas:

- **Why we can't guess probabilities (and shouldn't try).**
- **Info-gap models for severe uncertainties: in models, spatial distributions, pdfs.**
- **Example: robustness-premium for sub-optimal policy.**
- **Robustness is proxy for probability.**

¶ Questions:

- **Methods for policy selection with severe uncertainty?**
- **Is satisficing a last resort, or strategically advantageous?**

§ Info-gaps:

- **Incomplete understanding.**
- **Erroneous data.**
- **Changing conditions.**
- **Sur$_{\mathbf{p}}$rises.**

§ **Info-gaps: Surprises.**

§ **Fallacy of optimal model-based policy analysis.**

§ **Info-gaps: Surprises.**

§ **Fallacy of optimal model-based policy analysis.**

§ **Resolution: info-gap decision theory.**

- **Satisfice performance.**
- **Optimize robustness to uncertainty.**

2 *Principle of Indifference*

§ Question:

Is ignorance probabilistic?

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§ **Principle of indifference** (Bayes, LaPlace, Jaynes, . . .):

- **Elementary events,**
  about which nothing is known,
  are assigned equal probabilities.
- **uniform distribution represents**
  complete ignorance.

§ **The info-gap contention:**

The probabilistic domain of discourse
does not encompass all epistemic uncertainty.

2.1 *2-Envelope Riddle*

§ The riddle:

- **You are presented with two envelopes.**
  - **Each contains a positive sum of money.**
  - **One contains twice the contents of the other.**
- **You choose an envelope, open it, and find \$ 50.**
- **Would you like to switch envelopes?**

§ **You reason as follows:**

- **Other envelope contains either $ 25 or $ 100.**
- **Principle of indifference:**
- **Assume equal probabilities.**

  The expected utility upon switching is:

  E.U. $= \frac{1}{2}$ **$** $25 + \frac{1}{2}$ **$** $100 =$ **$** 62.50.

  $ $62.50 >$ **$** 50.
- **Yes! Let's switch, you say.**

§ The riddle, re-visited:

- **You are presented with two envelopes.**
  - **Each contains a positive sum of money.**
  - **One contains twice the contents of the other.**
- **You choose an envelope, but do not open it.**
- **Would you like to switch envelopes?**

§ You reason as follows:

- **This envelope contains $ $X > $ $ 0$.**
- **Other envelope contains either $ $2X$ or $ $\frac{1}{2}X$.**
- **Principle of indifference:**
- **Assume equal probabilities.**

  The expected utility upon switching is:

  E.U. $= \frac{1}{2}$ **$** $2X + \frac{1}{2}$ **$** $\frac{1}{2}X =$ **$** $\left(1 + \frac{1}{4}\right)X > X$.
- **Yes! Let's switch, you say.**

§ **You reason as follows:**

- **This envelope contains** $\$\, X > \$\, 0$.
- **Other envelope contains either** $\$\, 2X$ **or** $\$\, \frac{1}{2}X$.
- **Principle of indifference:**
- **Assume equal probabilities.**

  The expected utility upon switching is:

  $$\text{E.U.} = \frac{1}{2}\,\$\,2X + \frac{1}{2}\,\$\,\frac{1}{2}X = \$\left(1 + \frac{1}{4}\right)X > X.$$
- **Yes! Let's switch, you say.**

§ **You wanna switch again? And again? And again?**

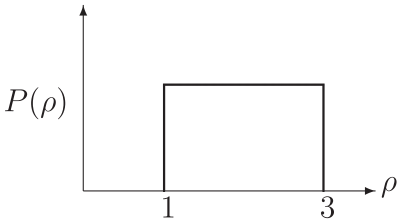
2.2 *Keynes' Example*

§ ρ = **specific gravity** [g/cm^3] **is unknown:**

$$1 \ \leq \ \rho \ \leq \ 3$$

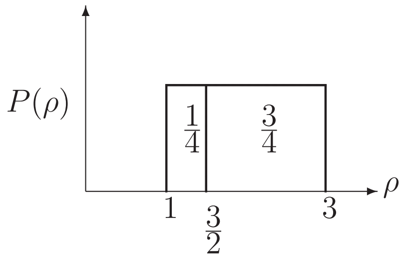
§ **Principle of indifference:**

Uniform distribution in $[1, \ 3]$**, so:**



§ Uniform distribution in $[1,\ 3]$, so:

$$\mathbf{Prob}\left(\frac{3}{2} \ \leq \ \rho \ \leq \ 3\right) = \frac{3}{4}$$

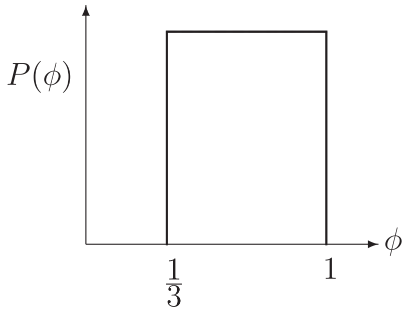


§ ϕ = specific volume [cm^3/g] is unknown:

$$\frac{1}{3} \;\leq\; \phi \;\leq\; 1$$

§ Principle of indifference:

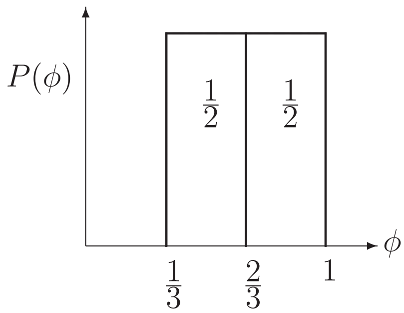
Uniform distribution in $\left[\frac{1}{3},\ 1\right]$, so:



§ Principle of indifference:

Uniform distribution in $\left[\frac{1}{3},\ 1\right]$, so:

$$\textbf{Prob}\left(\frac{1}{3}\ \le\ \phi\ \le\ \frac{2}{3}\right) = \frac{1}{2}$$



§ Contradiction:

$$\frac{1}{2} = \underbrace{\mathbf{Prob}\left(\frac{1}{3} \;\leq\; \phi \;\leq\; \frac{2}{3}\right)}_{\textbf{Specific volume}} = \underbrace{\mathbf{Prob}\left(\frac{3}{2} \;\leq\; \rho \;\leq\; 3\right)}_{\textbf{Specific gravity}} = \frac{3}{4}$$

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§ The Culprit:

- **Principle of indifference.**

§ The resolution:

- **Ignorance is not probabilistic.**
- **Ignorance is an info-gap.**

2.3 *Shackle-Popper Indeterminism*

§ **Intelligence:**

What people know,

influences how they behave.

§ **Discovery:**

What will be discovered tomorrow

cannot be known today.

§ **Indeterminism:**

Tomorrow's behavior cannot be

modelled completely today.

§ **Information-gaps, indeterminisms, sometimes cannot be modelled probabilistically.**

§ **Ignorance is not probabilistic.**

3 *POLICY SELECTION: SIMPLE EXAMPLE*

§ The problem: Surprises!

- **"The true art of good monetary policy is in managing forecast surprises." (William Poole, 2004)**
- **"Optimization works in theory but risk management is better in practice. There is no scientific way to compute an optimal path for monetary policy." (Alan Greenspan, 2005)**

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§ The solution: Caution.

§ Our method: Info-gap robust-satisficing.

§ System model:

$$Y = GP + Z$$

Y= **Variable to be controlled.**

P= **Policy variable to be chosen.**

G, Z= **Highly uncertain model variables.**

§ Info-gap uncertainty in G, Z:

- **Known typical values, $\widetilde{G}$, $\widetilde{Z}$.**
- **Unknown range of error: surprises.**
- **Unknown probability distribution.**
- **Unknown fractional error info-gap model:**

$$\mathcal{U}(\alpha, \widetilde{G}, \widetilde{Z}) = \left\{ G, Z : \ \left| \frac{G - \widetilde{G}}{\widetilde{G}} \right| \le \alpha, \ \left| \frac{Z - \widetilde{Z}}{\widetilde{Z}} \right| \le \alpha \right\}, \quad \alpha \ge 0$$

α = Unknown horizon of uncertainty.

§ Another info-gap model:

Estimated PDF with uncertain tails.

§ Performance function:

$$E^2(P,G,Z) = \left[Y(P,G,Z) - Y^\bullet\right]^2$$

$Y^\bullet$ = **Target value.**

§ Satisfice performance:

$$E(P,G,Z) \le E_{\rm c}$$

§ Robustness questions:

- **How wrong can the model be ($\widetilde{G}$, $\widetilde{Z}$) w/o jeopardizing performance ($E \le E_{\rm c}$)?**
- **What performance can be reliably anticipated?**
- **Policy implication?**

§ Robustness:

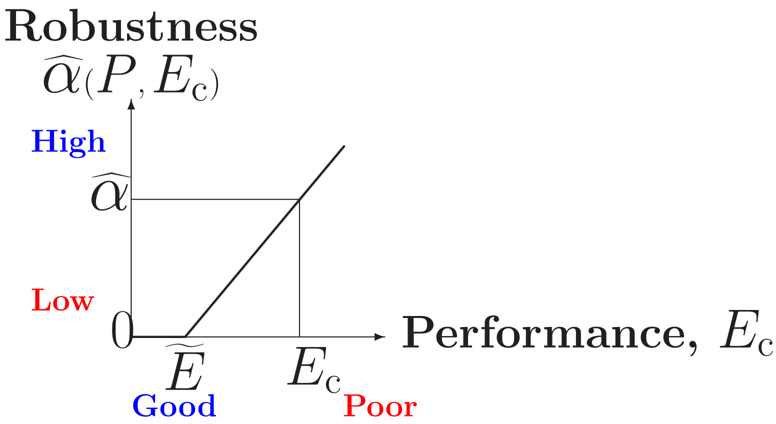
- $\mathcal{U}(\alpha, \widetilde{G}, \widetilde{Z})$ = **info-gap uncertainty.**
- α = **unknown horizon of uncertainty.**
- **Robustness = max. tolerable α:**

$$\widehat{\alpha}(P, E_{\rm c}) = \max\left\{\alpha : \ \left(\max_{G,Z \in \mathcal{U}(\alpha,\widetilde{G},\widetilde{Z})} E(P,G,Z)\right) \le E_{\rm c}\right\}$$

§ Preferences:

$$P \succ P^{\bullet} \quad \textbf{if} \quad \widehat{\alpha}(P, E_{\rm c}) > \widehat{\alpha}(P^{\bullet}, E_{\rm c})$$

- **Satisfice performance at $E_{\rm c}$.**
- **Maximize robustness $\widehat{\alpha}(P, E_{\rm c})$.**



§ **Trade-off: robustness vs. performance.**

§ **Anticipated outcome:** $\widetilde{E} = E(P, \widetilde{G}, \widetilde{Z})$

Zero robustness.

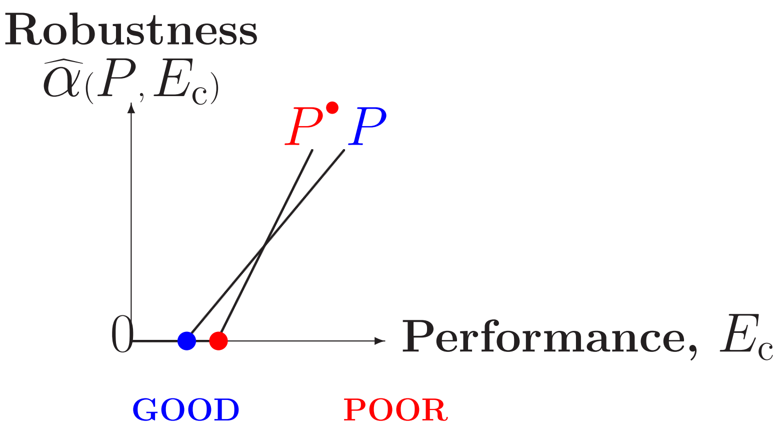
§ **Sub-optimal outcome:** $E_c > E(P, \widetilde{G}, \widetilde{Z})$

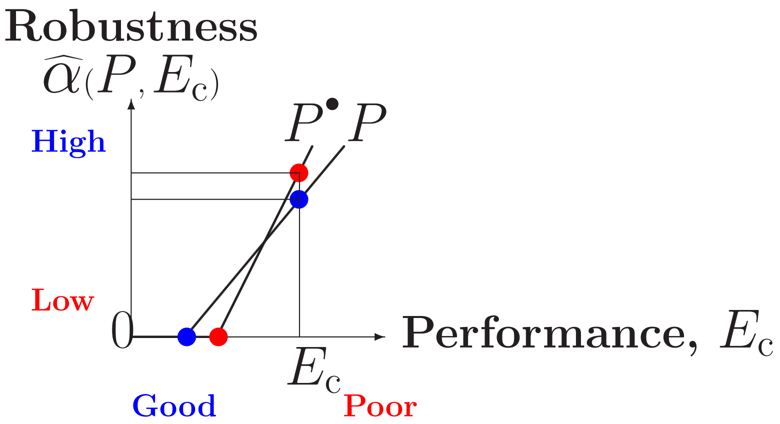
Positive robustness.

§ Two policy choices, P, $P^{\bullet}$:

Best-model preference:

$$E(P,\widetilde{G},\widetilde{Z}) < E(P^{\bullet},\widetilde{G},\widetilde{Z}) \quad \Longrightarrow \quad P \succ P^{\bullet}$$





§ Preference reversal:

- **Best-model preference:**

$$E(P, \widetilde{G}, \widetilde{Z}) < E(P^\bullet, \widetilde{G}, \widetilde{Z}) \quad \Longrightarrow \quad P \succ P^\bullet$$

- **Robust preference at E_c:**

$$\widehat{\alpha}(P^\bullet, E_c) > \widehat{\alpha}(P, E_c) \quad \Longrightarrow \quad P \prec P^\bullet$$

- **Calculate optimum (P)**

  but (possibly) do something else ($P^\bullet$).

§ Robustness & probability of policy success

- **Model:** $Y = GP + Z$.
- **Use policy P to reach target $Y^{\bullet}$.**
- **Error:** $E^2(G, Z) = (Y - Y^{\bullet})^2$.
- **Require:** $E(G, Z) \le E_{\mathrm{c}}$.
- (G, Z) **uncertain:**
  - $\mathcal{U}(\alpha, \widetilde{G}, \widetilde{Z})$ = **known info-gap model.**
  - $F(G, Z)$ = **unknown prob. distribution.**
- **Probability of policy success:**

$$P_{\mathrm{s}}(P) = F[E(G, Z) \le E_{\mathrm{c}}]$$

- **Probability of policy success:**

$$P_{\rm s}(P) = F[E(G,Z) \le E_{\rm c}]$$

- $P_{\rm s}(P)$ **unknown.** $\widehat{\alpha}(P, E_{\rm c})$ **known.**

§ **"Theorems":**

$$\left(\frac{\partial \widehat{\alpha}(P, E_{\rm c})}{\partial P} \right) \left(\frac{\partial P_{\rm s}(P)}{\partial P} \right) \ge 0$$

- **Robustness proxies for prob. of success.**
- **We can't calculate** $P_{\rm s}(P)$**.**
- **We can calculate** $\widehat{\alpha}(P, E_{\rm c})$**, so**
- **We can maximize** $P_{\rm s}(P)$**.**

4 *SUMMARY*

§ **Model-based policy analysis:**

Adapt policy to attributes of model.

§ **Optimization:**

Use best model to

choose policy with best outcome.

§ Uncertainty:

- **Randomness:** **structured uncertainty.**
- **Info-gaps:** **surprises, ignorance.**

§ Fallacy of optimal model-based policy:

- **Severe uncertainty:**
  - **Best model errs seriously.**
  - **Some model attributes are correct.**
  - **Some model attributes err greatly.**
- **Best-model optimization**
  - **exploits all model attributes to extreme.**
  - **Vulnerable to model error.**

§ Resolution: Info-gap decision theory.

- **Satisfice performance.**
- **Optimize robustness to uncertainty.**
- **Model and manage sur^p^rises.**

§ Robust-satisficing syllogism:

- **Adequate performance is necessary.**
- **More reliable adequate performance is better than less reliable adeq. perf.**
- **Thus maximum reliability is best.**

§ **Robustness-premium theorem:**

Sub-optimal policy has

greater robustness to uncertainty

than best-model optimal policy.

¶ **Robustness-proxy theorems:**

Sub-optimal policy has

greater probability of success

than best-model optimal policy.