

Info-Gap Methods

for

Decision Support

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1 *HIGHLIGHTS*

§ Info-gaps:

- Incomplete understanding.
- Erroneous data.
- Changing conditions.
- Sur_prises.

§ Info-gaps:

- Incomplete understanding.
- Erroneous data.
- Changing conditions.
- Sur_pprises.

¶ Questions:

- Updating.
- Decision making.
- Forecasting.

2 *BIOLOGICAL CONSERVATION: EXAMPLES*

2.1 Protection Against Infestation

§ **Danger:** Infestation at any point x .

§ **Resources:** q .

§ **Uncertainties:**

- Prob. of infestation at x : $p(x)$.
- Loss from infestation at x : $L(x, q)$.

§ **Assessment:** Expected loss:

$$E(L|p, q) = \int L(x, q)p(x) dx$$

§ Tasks:

- Allocate resources.
- Satisfice expected loss.
- Maximize robustness to uncertainty.

§ Extensions:

- Strategic opponent, $p(x, q)$.
- Temporal dynamics, $p(x, t)$.

2.2 Spatial Monitoring

§ **Danger:** Extreme toxin.

§ **Resources:** Measure at N sites.

§ **Uncertainty:** Spatial distribution of toxin.

§ **Remedial action** if
critical toxin level detected.

§ **Tasks:**

- Choose #, place, type of measurements.
- Satisfice level detected.
- Max. robustness to spatial uncertainty.

§ Extensions:

- Multiple toxins.
- Latent toxins.
- New (uncertain) toxins.
- Transport dynamics (uncertain).
- Probabilistic detection.

2.3 Investment for Bio-Diversity

§ **Goal:** Enhance bio-diversity.

§ **Method:** Invest in land, treatment, etc.

§ **Problems:**

- Limited resources, q .
- Uncertain bio-div return, $N(q, t)$.
- Returns and resources emerge (uncertainly) over time.

2.4 Statistical Tests with Distributional Uncertainty

§ Goal:

Determine freq. of occurrence of species.

§ Data: Non-random, heterogeneous data.

E.g. Various observer talent, effort, sites.

§ Method: Statistical test, e.g. t , χ^2 , etc.

§ Problem: Distributional uncertainty.

3 *Setting Conservation Priorities For Land Rehabilitation*

3.1 **Models and Uncertainties**

§ **The goal:**

- **Bio-diversity and land management.**
- **Enhance number of species by
joining remnant land to new patch
at specified distance.**

§ **The problem: Uncertain model and data.**

§ References:

- Moilanen, Runge, Elith, Tyre, Carmel, Fegraus, Wintle, Burgman and Ben-Haim, **Planning for robust reserve networks using uncertainty analysis**, *Ecological Modelling*, 2006.
- Moilanen, van Teeffelen, Ben-Haim and Ferrier, **How much compensation is enough? A framework for incorporating uncertainty and time discounting when calculating offset ratios for impacted habitat**, *Restoration Ecology*, 2008.

§ Ecological model:

Increments in # of species:

$$\text{Remnant : } S'_1 - S_1 = \left[(R_1 + A_0)^Z - R_1^Z \right] e^{-\beta D} C$$

$$\text{Patch : } S'_0 - S_0 = \left[(R_1 + A_0)^Z - A_0^Z \right] e^{-\beta D} C$$

§ Decision variables: $Q = (R_1, A_0, D)$:

R_1 = remnant area.

A_0 = patch area.

D = distance from remnant to patch.

§ Uncertain variables: $U = (\beta, C, Z)$.

§ **Performance function: # of new species:**

$$N(Q, U) = \max[S'_0 - S_0, S'_1 - S_1]$$

§ **Performance requirement:**

Achieve at least minimal growth.

$$N(Q, U) \geq N_C$$

§ Info-gap model of uncertainty:

- **Uncertain model parameters:** $U = (\beta, C, Z)$.
- **Best estimates:** $\tilde{U} = (\tilde{\beta}, \tilde{C}, \tilde{Z})$.
- **Unknown fractional errors of estimates:**

$$\left| \frac{U_I - \tilde{U}_I}{\tilde{U}_I} \right| \leq \alpha, \quad \alpha \geq 0$$

- **Info-gap model:**

$$\mathcal{U}(\alpha, \tilde{U}) = \left\{ U : \left| \frac{U_I - \tilde{U}_I}{\tilde{U}_I} \right| \leq \alpha, \quad I = 1, 2, 3 \right\}, \quad \alpha \geq 0$$

Unbounded family of nested sets.

§ Many info-gap models of uncertainty:

$$u(\alpha, \bar{U}) = \{U \in \mathfrak{R}^N : (U - \bar{U})^T W (U - \bar{U}) \leq \alpha^2\}, \quad \alpha \geq 0$$

$$u(\alpha, \bar{U}) = \{U(T) : \int [U(T) - \bar{U}(T)]^2 dT \leq \alpha^2\}, \quad \alpha \geq 0$$

$$u(\alpha, \bar{P}) = \left\{ P(T) : P(T) \in \mathcal{P}, \left| \frac{P(T) - \bar{P}(T)}{\bar{P}(T)} \right| \leq \alpha \right\}, \quad \alpha \geq 0$$

Many more ...

§ Axioms of info-gap models of uncertainty:

Contraction: $\mathcal{U}(0, \bar{u}) = \{\bar{u}\}$

Nesting: $\alpha < \alpha^\bullet \implies \mathcal{U}(\alpha, \bar{u}) \subset \mathcal{U}(\alpha^\bullet, \bar{u})$

α is unknown horizon of uncertainty.

3.2 Robustness to Uncertainty

§ Robustness of design Q :

Max horizon of uncertainty, α ,

at which all realizations U

satisfy growth requirement, $N(Q, U) \geq N_C$.

$$\bar{\alpha}(Q, N_C) = \max \left\{ \alpha : \left(\min_{U \in \mathcal{U}(\alpha, \bar{U})} N(Q, U) \right) \geq N_C \right\}$$

§ Preferences.

Robustness: bigger is better:

$$Q \succ Q^\bullet \quad \text{if} \quad \bar{\alpha}(Q, N_C) > \bar{\alpha}(Q^\bullet, N_C)$$

§ Decision strategy: Robust satisficing.

- Satisfy performance requirements.
- Maximize robustness to uncertainty.

$$\widehat{Q}(N_C) = \arg \max_Q \widehat{\alpha}(Q, N_C)$$

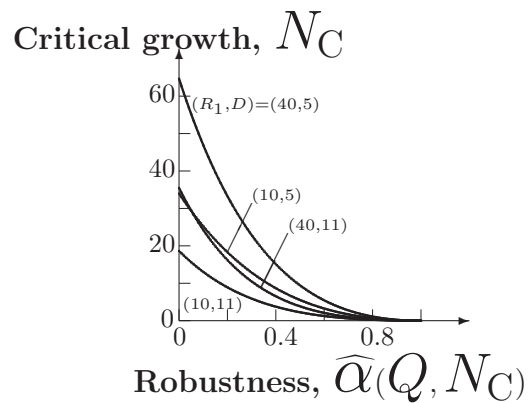
§ Robust-satisficing syllogism:

- Adequate performance is necessary.
- More reliable adequate performance is better than less reliable adeq. perf.
- Thus maximum reliability is best.

§ Trade-off:

$N_C > N_C^\bullet$ implies $\bar{\alpha}(Q, N_C) < \bar{\alpha}(Q, N_C^\bullet)$

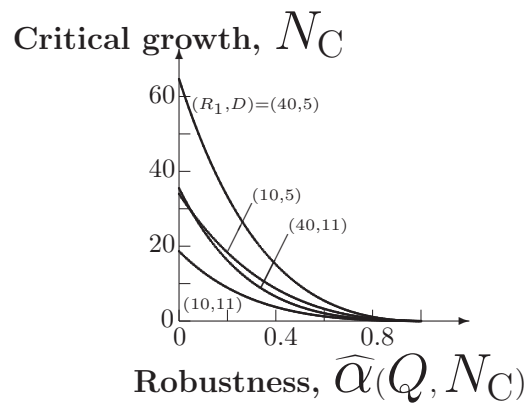
- More demanding performance:
 - Less immunity to uncertainty.
 - Less reliable achievement.



§ Why not optimize with best model?

$$N_C = N(Q, \widetilde{U}) \quad \text{implies} \quad \widehat{\alpha}(Q, N_C) = 0$$

- Performance of best model has zero robustness to model error.
- Only lower aspirations have higher robustness.



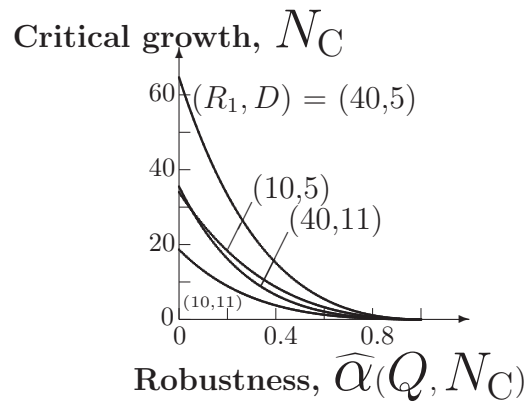
§ Why not optimize with best model?

Best-model optimum:

$$Q^* = \arg \max_Q N(Q, \tilde{U})$$

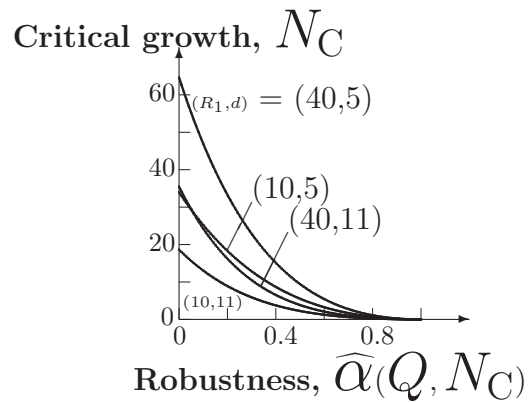
has zero robustness:

$$N_C = N(Q^*, \tilde{U}) \quad \text{implies} \quad \bar{\alpha}(Q^*, N_C) = 0$$



§ **Decisions:** Choose area R_1 , distance D .
 $(A_0 = 1)$

- Large R_1 better than small R_1 : $40 \succ 10$.
- Small D better than large D : $5 \succ 11$.
- R_1 trades-off against D : $(10, 5) \approx (40, 11)$.



§ What does robustness mean?

- $\hat{\alpha}(Q, N_C) = 0.4$:
 $N(Q, U) \geq N_C$ is guaranteed
 for model error $\leq \pm 40\%$.
- $\hat{\alpha}(Q, 15) = 0.4$ for $Q = (R_1, A_0, D) = (40, 1, 5)$.
- $\hat{\alpha}(Q, 3) = 0.4$ for $Q = (R_1, A_0, D) = (10, 1, 11)$.

3.3 Opportuneness from Uncertainty

§ Two faces of uncertainty:

- **Pernicious.**
- **Propitious.**

§ Two decision strategies:

- **Robust satisficing:**
 - Satisfice the performance.
 - Maximize robustness against pernicious uncertainty.
- **Opportune windfalling:**
 - Windfall the performance.
 - Minimize robustness against propitious uncertainty.

§ Windfalling:

- $N_W =$ Large number of new species.
- $N_W \gg N_C$.
- N_C is critical, necessary.
- N_W is wonderful windfall!

§ Opportuneness of design Q :

Min horizon of uncertainty, α ,

at which at least one realization U

enables windfall: $N(Q, U) \geq N_W$.

$$\widehat{\beta}(Q, N_W) = \min \left\{ \alpha : \left(\max_{U \in \mathcal{U}(\alpha, \widetilde{U})} N(Q, U) \right) \geq N_W \right\}$$

§ Robustness and opportuneness: duality

$$\bar{\alpha}(Q, N_C) = \max \left\{ \alpha : \left(\min_{U \in \mathcal{U}(\alpha, \tilde{U})} N(Q, U) \right) \geq N_C \right\}$$

$$\hat{\beta}(Q, N_W) = \min \left\{ \alpha : \left(\max_{U \in \mathcal{U}(\alpha, \tilde{U})} N(Q, U) \right) \geq N_W \right\}$$

§ Preferences.

- **Robustness: bigger is better.**

$$Q \succ Q^\bullet \quad \text{if} \quad \widehat{\alpha}(Q, N_C) > \widehat{\alpha}(Q^\bullet, N_C)$$

- **Opportuneness: big is bad.**

$$Q \succ Q^\bullet \quad \text{if} \quad \widehat{\beta}(Q, N_W) < \widehat{\beta}(Q^\bullet, N_W)$$

- **Antagonism or sympathy.**

§ Decision strategy: opportune windfalling.

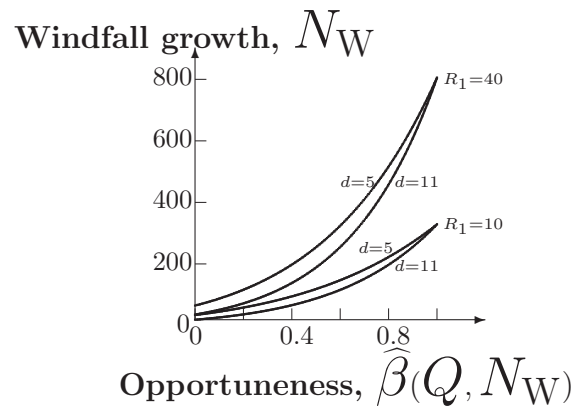
- Windfall the performance requirements.
- Minimize robustness to propitious uncertainty.

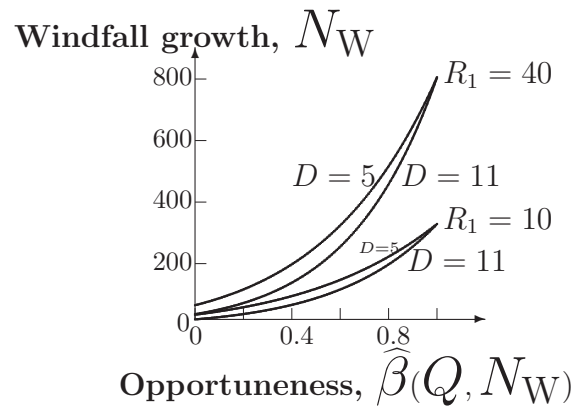
$$\widehat{Q}(N_W) = \arg \min_Q \widehat{\beta}(Q, N_W)$$

§ Trade-off:

$N_W > N_W^\bullet$ implies $\widehat{\beta}(Q, N_W) > \widehat{\beta}(Q, N_W^\bullet)$

- Wilder windfall performance requires more uncertainty.





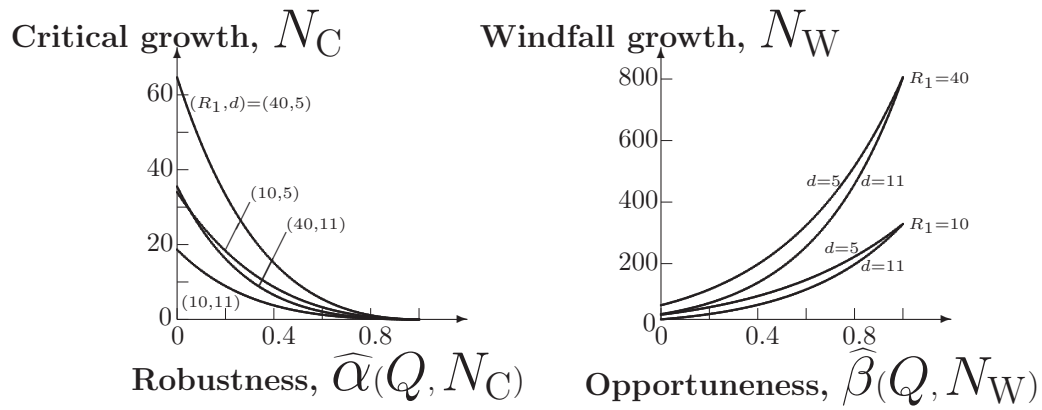
§ What does opportuneness mean?

- $\hat{\beta}(Q, N_W) = 0.4$:

$N(Q, U) \geq N_W$ is possible

only if model errors $\geq \pm 40\%$.

- $\hat{\beta}(Q, 200) = 0.4$ for $Q = (R_1, A_0, D) = (40, 1, 5)$.
- $\hat{\beta}(Q, 200) = 0.75$ for $Q = (R_1, A_0, D) = (10, 1, 5)$.



§ Robustness and opportuneness:

- Performance range:

$$\widehat{\alpha}(Q, 15) = 0.4 = \widehat{\beta}(Q, 200)$$

for $Q = (R_1, A_0, D) = (40, 1, 5)$.

- Sympathetic immunities.

3.4 Robustness and Probability of Design Success

§ Probability of design success

- **Model:** $N(Q, U)$.
- Use **design** Q to reach **target** N_C .
- **Require:** $N(Q, U) \geq N_C$.
- U uncertain:
 - $\mathcal{U}(\alpha, U) =$ **known** info-gap model.
 - $F(U) =$ **unknown** prob. distribution.
- **Probability of design success:**

$$P_s(Q, N_C) = F[N(Q, U) \geq N_C]$$

§ “Theorems”:

$$\left(\frac{\partial \widehat{\alpha}(Q, N_C)}{\partial Q} \right) \left(\frac{\partial P_s(Q, N_C)}{\partial Q} \right) \geq 0$$

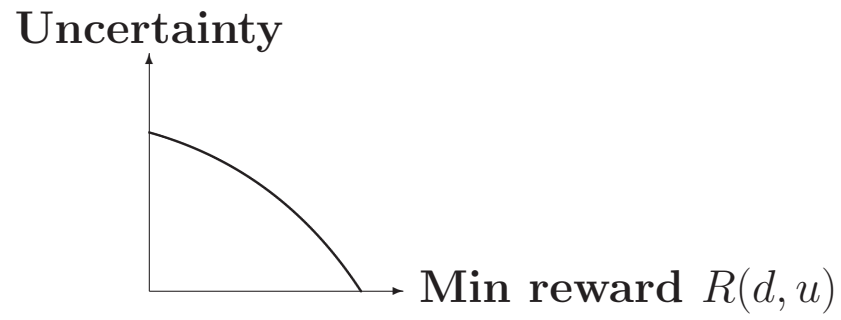
- **Robustness proxies for prob. of success.**
- We **can't** calculate $P_s(Q, N_C)$.
- We **can** calculate $\widehat{\alpha}(Q, N_C)$, **so**
- We **can** maximize $P_s(Q, N_C)$.

4 *MAX-MIN and ROBUST-SATISFICING*

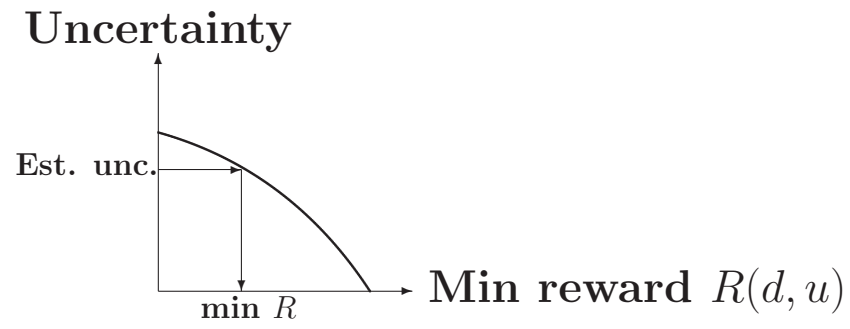
§ **Task:** make a decision.

- $d =$ decision.
- $u =$ uncertain parameters, functs., sets.
- $R(d, u) =$ reward.

§ Trade-off: uncertainty vs. min reward.

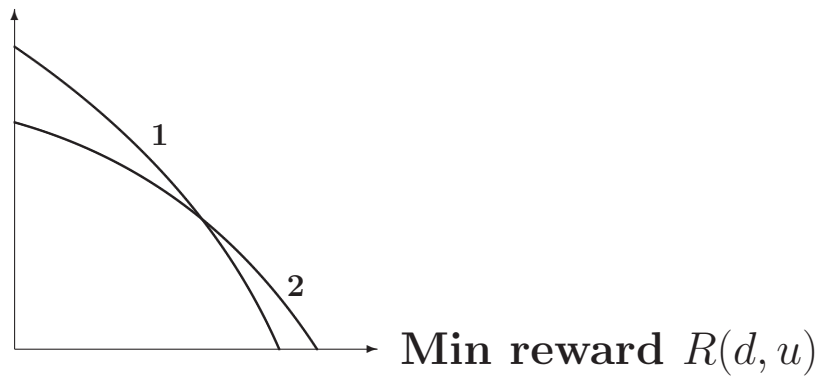


§ Trade-off: uncertainty vs. min reward.

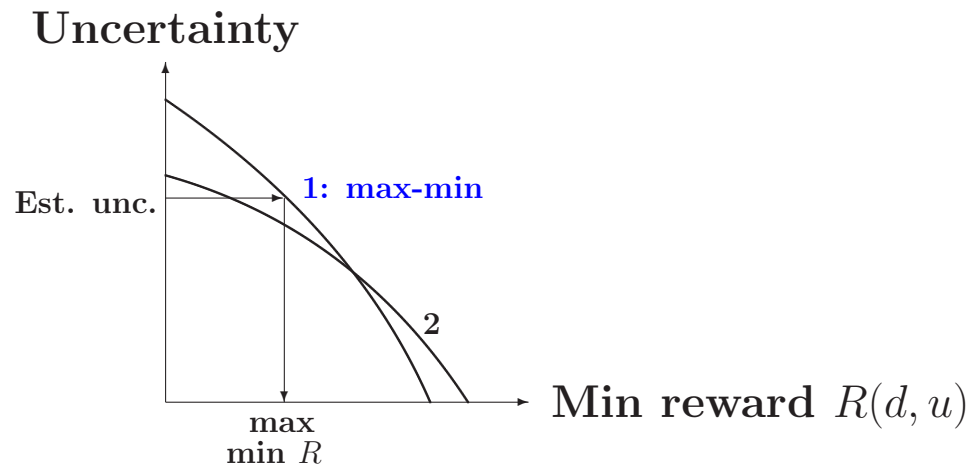


§ Choose from 2 decisions: d_1, d_2 .

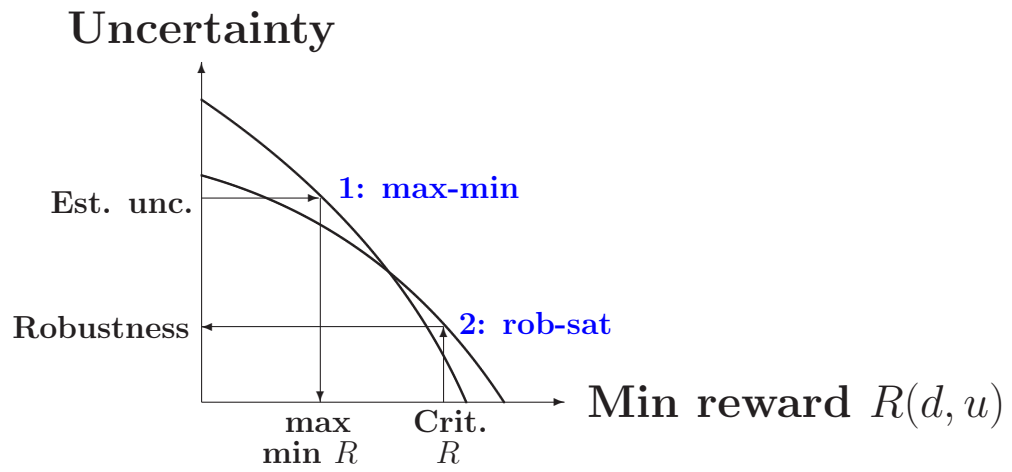
Uncertainty



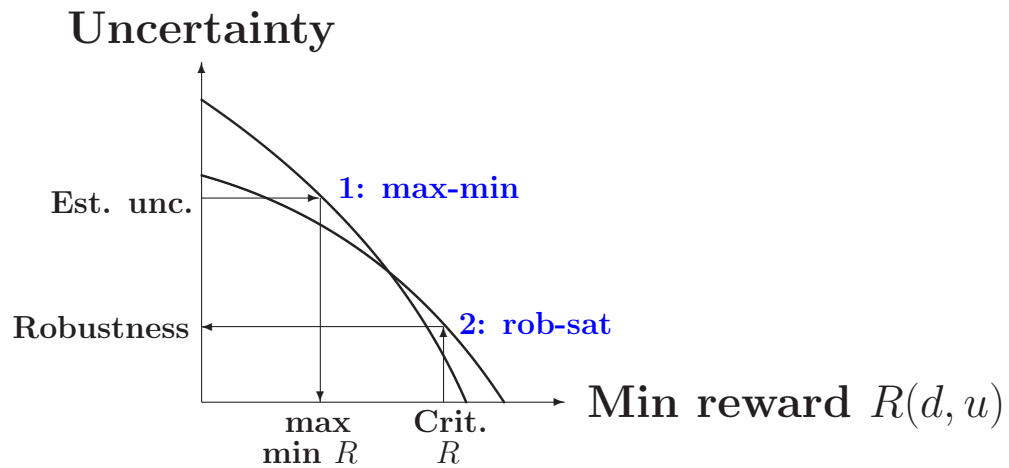
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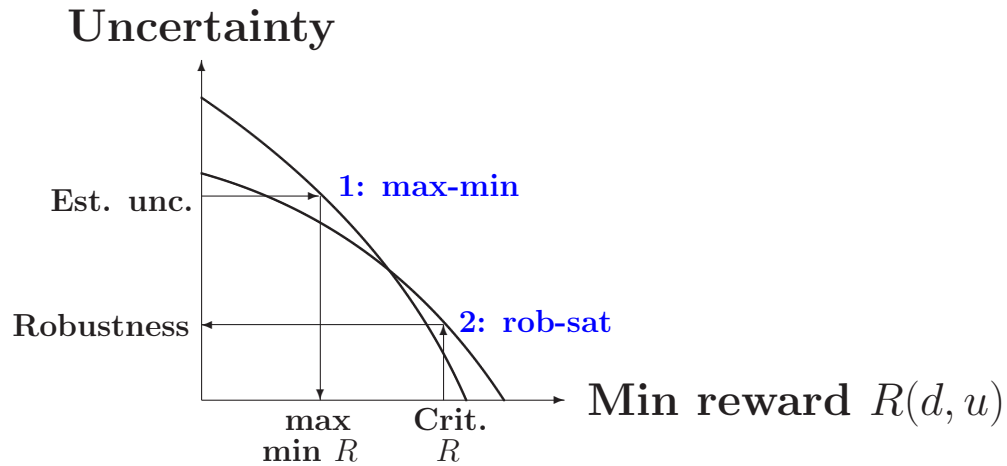


§ Modeller's equivalence:

Max-min can always **describe** rob-sat.

(by adjusting prior beliefs)

§ Choose from 2 decisions: d_1, d_2 .



§ Modeller's equivalence:

Max-min can always **describe** rob-sat.
(by adjusting prior beliefs)

§ Decision-maker's duality:

Max-min and rob-sat **differ** if:

- Max-min gain too low, or,
- Worst case is uncertain.

§ Evolutionary advantage of robustness:

Robustness is a **proxy for**
Probability of survival.

§ Evolutionary advantage of robustness:

Robustness is a **proxy for**
Probability of survival.

§ **Survival:** $R(d, u) \geq R_c$.

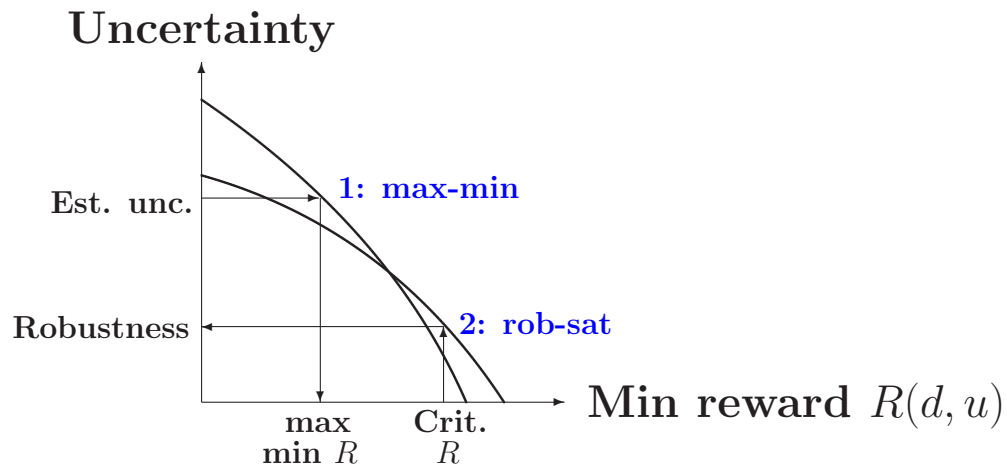
§ **Probability of survival:** $P_s(d, R_c) = \int_{R \geq R_c} p(u) du$

§ **Robustness:** $\widehat{\alpha}(d, R_c)$.

§ **Proxy Theorem:**

$$\left(\frac{\partial \widehat{\alpha}(d, u)}{\partial d} \right) \left(\frac{\partial P_s(d, u)}{\partial d} \right) \geq 0$$

§ Choose from 2 decisions: d_1, d_2 .



§ Modeller's equivalence:

Max-min can always describe rob-sat.

§ Decision-maker's duality:

Max-min and rob-sat differ if:

- Max-min gain too low, or,
- Worst case is uncertain.

§ Rob-sat beats max-min in competition.

5 FAULT DETECTION

§ The goal:

Distinguish between

No Fail (NF) and **Fail** (F).

§ Use vector measurement x .

- Declare **NF** if $x \in D$.
- Declare **F** if $x \notin D$.

§ Choose D for:

- Small probability of missed detection.
- Small probability of false alarm.

§ Uncertain pdfs:

- **Estimated:** **Actual:**
 - **NF:** $\pi_0(x)$. $p_0(x)$.
 - **F:** $\pi_1(x)$. $p_1(x)$.
- **Info-gap models:**
 - **NF:** $\mathcal{U}_0(\alpha, \pi_0)$, $\alpha \geq 0$.
 - **F:** $\mathcal{U}_1(\alpha, \pi_1)$, $\alpha \geq 0$.

Unbounded families of nested sets of pdfs.

- **E.g. fractional-error info-gap model:**

$$\mathcal{U}_i(\alpha, \pi_i) = \{p(x) : p(x) \in \mathcal{P}_i, |p(x) - \pi_i(x)| \leq \alpha\pi_i(x)\}, \alpha \geq 0$$

Unbounded uncertainty. No worst case.

§ Performance functions:

- Probability of missed detection:

$$P_0(p) = \Pr(x \in D|F)$$

- Probability of false alarm:

$$P_1(p) = \Pr(x \notin D|NF)$$

§ Performance spec. Choose D so that:

- $P_0(p) \leq P_{0c}$
- $P_1(p) \leq P_{1c}$.

§ The problem: $p(x)$ highly uncertain.

§ Info-gap robust-satisficing:

- Given specs P_{0c} and P_{1c} choose D to:
- Satisfice performance.
- Maximize robustness.

§ Robustness to $\mathcal{U}_0(\alpha, \pi_0)$, $\mathcal{U}_1(\alpha, \pi_1)$, $\alpha \geq 0$:

$$\widehat{\alpha}(D, P_{0c}, P_{1c}) = \max \left\{ \alpha : \begin{array}{l} \left(\max_{p \in \mathcal{U}_1(\alpha, \pi_1)} P_0(p) \right) \leq P_{0c}, \\ \left(\max_{p \in \mathcal{U}_0(\alpha, \pi_0)} P_1(p) \right) \leq P_{1c} \end{array} \right\}$$

§ Robust-satisficing design:

$$\widehat{D}(P_{0c}, P_{1c}) = \arg \max_D \widehat{\alpha}(D, P_{0c}, P_{1c})$$

§ Trade-off:

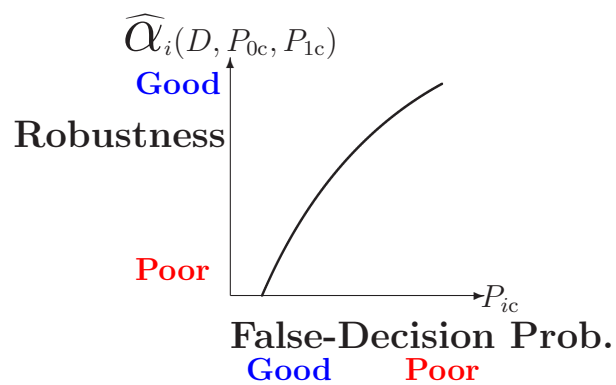
- Better perf. \iff Lower robustness:

- Missed-detection performance trade-off:

$$P_{0c} < P'_{0c} \text{ implies } \widehat{\alpha}(D, P_{0c}, P_{1c}) \leq \widehat{\alpha}(D, P'_{0c}, P_{1c})$$

- False-alarm performance trade-off:

$$P_{1c} < P'_{1c} \text{ implies } \widehat{\alpha}(D, P_{0c}, P_{1c}) \leq \widehat{\alpha}(D, P_{0c}, P'_{1c})$$



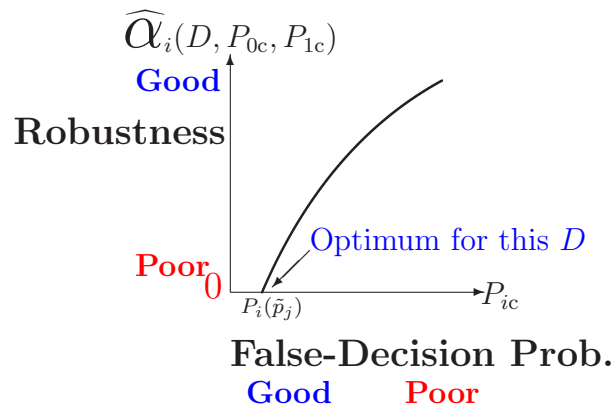
§ Trade-off:

• Better perf. \iff Lower robustness.

• **Estimated perf. has zero robustness:**

$$\widehat{\alpha}(D, P_{0c}, P_{1c}) = 0 \quad \text{if } P_{0c} = P_0(\pi_1) \quad \text{or if } P_{1c} = P_1(\pi_0)$$

• **Pareto optimum has zero robustness.**



§ Robust-satisficing may differ from Pareto optimization.

§ Pareto optimal design, D^* :

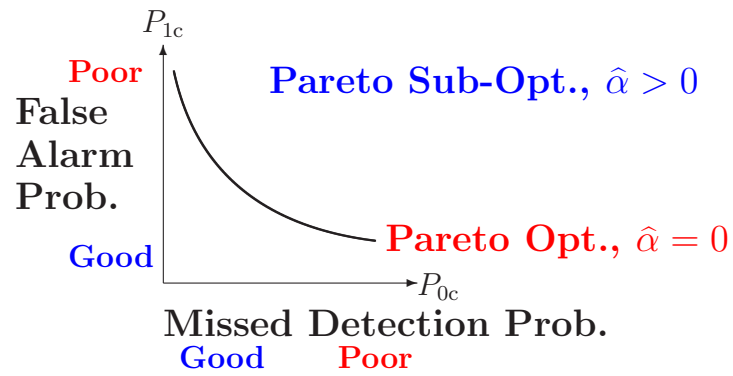
Any change **reducing** one failure prob. **increases** the other.

§ Estimated error probabilities with D^* :

$$P_0^* = P_0(\pi_1, D^*), \quad P_1^* = P_1(\pi_0, D^*)$$

No robustness of estim. Pareto optimum:

$$\widehat{\alpha}(D^*, P_0^*, P_1^*) = 0$$



§ Estimated error probabilities with D^* :

$$P_0^* = P_0(\pi_1, D^*), \quad P_1^* = P_1(\pi_0, D^*)$$

- **No robustness of Pareto optima:**

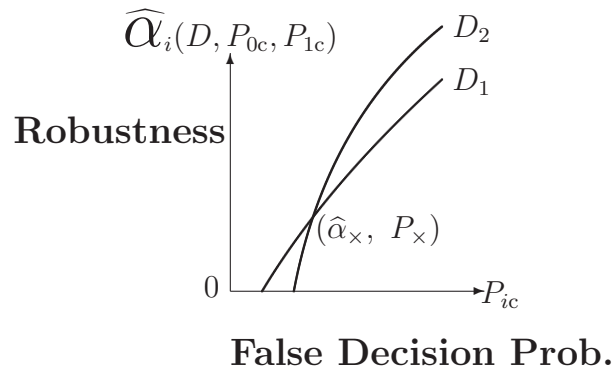
$$\widehat{\alpha}(D^*, P_0^*, P_1^*) = 0$$

- **Positive robustness of Pareto sub-optima:**

$$\widehat{\alpha}(D, P_{0c}, P_{1c}) > 0$$

Preference Reversal

§ Two designs: D_1 and D_2 :



§ Preferences:

- Nominal: $D_1 \succ D_2$.
- Low robustness: $D_1 \succ D_2$.
- High robustness: $D_2 \succ D_1$.

Example

§ Estimated Exponentials:

$$\pi_i(x) = \lambda_i e^{-\lambda_i x}, \quad i = 0, 1$$

$$\lambda_0 > \lambda_1. \quad \text{NF: } 0. \quad \text{F: } 1.$$

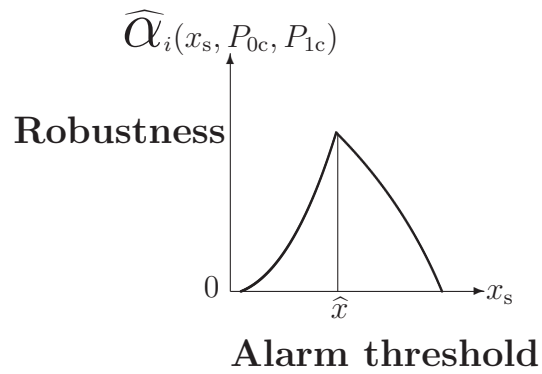
§ NF decision region:

$$D = [0, x_s]$$

§ Choose x_s to:

- Satisfice performance.
- Maximize robustness.

$$\hat{x}(P_{0c}, P_{1c}) = \arg \max_{x_s} \widehat{\alpha}(x_s, P_{0c}, P_{1c})$$



Optimize or Robust-Satisfice?

§ Optimization syllogism:

- Outcome is valuable.
- More value is better than less value.
- Most valuable outcome is best.

§ Implicit optimization syllogism:

- Outcome values are uncertain.
- High value safer than low value.
- Highest value safest.

§ Robust-satisficing syllogism:

- Critical value necessary for survival.
- High reliability of critical value better than low reliability.
- Max reliability of critical value is best.

Why Robustness?

§ Adequate error probabilities:

$$P_0(p) = \text{missed detection prob.} \leq P_{0c}.$$

$$P_1(p) = \text{false alarm probability} \leq P_{1c}.$$

for greater range of pdfs.

§ More confidence in outcome.

§ Greater probability of success.

Probability of Successful Detection

§ $p(x)$ random function; distribution $\Pi(p)$.

§ $p(x)$ **info-gap-uncertain.**

§ **Probability of success:**

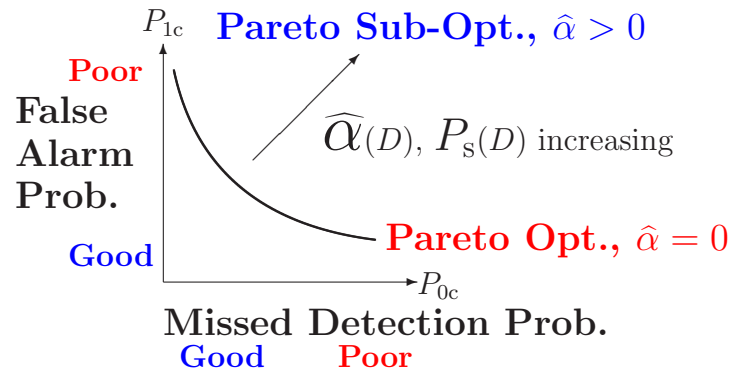
$$P_s(D) = \Pi(P_0(p) \leq P_{0c}, P_1(p) \leq P_{1c})$$

§ **Theorems. Robustness and success prob:**

$$\frac{d\widehat{\alpha}(D, P_{0c}, P_{1c})}{dD} \geq 0 \quad \text{if and only if} \quad \frac{dP_s(D)}{dD} \geq 0$$

§ Theorems. Robustness and success prob:

$$\frac{d\widehat{\alpha}(D, P_{0c}, P_{1c})}{dD} \geq 0 \quad \text{if and only if} \quad \frac{dP_s(D)}{dD} \geq 0$$



Summary

§ Models (pdfs):

Attributes of model correspond to attributes of reality.

§ Model-based design:

Adapt design, D , to attributes of model.

§ Optimization:

Use best model to choose design with best outcome.

§ Uncertainty:

- **Randomness:** structured uncertainty.
- **Info-gaps:** surprises, ignorance.

§ **Fallacy** of optimal model-based design:

- **Severe uncertainty:**
 - Best models err seriously.
 - Some model attributes are **correct**.
 - Some model attributes **err greatly**.
- **Best-model optimization**
 - exploits **all model attributes** to extreme.
 - Vulnerable to model error.

§ Resolution: Info-gap decision theory.

- Satisfice performance.
- Maximize robustness to uncertainty.
- Model and manage sur^Prises.

§ Robust-satisficing syllogism:

- Adequate performance is necessary.
- More reliable adequate performance is better than less reliable adeq. perf.
- Thus maximum reliability is best.

§ Theorems:

- Positive robustness only for Pareto sub-optima.
- Increasing probability of success with increasing robustness.

6 *SUMMARY*

§ **Models:**

Attributes of model correspond to attributes of reality.

§ **Model-based decision:**

Adapt decision to attributes of model.

§ **Optimization:**

Use best model to choose decision with best outcome.

§ Uncertainty:

- **Randomness:** structured uncertainty.
- **Info-gaps:** surprises, ignorance.

§ **Fallacy of optimal model-based decision:**

- **Severe uncertainty:**
 - Best model errs seriously.
 - Some model attributes are **correct**.
 - Some model attributes **err greatly**.
- **Best-model optimization**
 - exploits **all model attributes** to extreme.
 - Vulnerable to model error.

§ Resolution: Info-gap decision theory.

- Robust-satisficing.
- Opportune-windfalling.
- Model and manage sur^Prises.

§ Proxy theorems:

max robustness \equiv max survival prob.

§ Applications:

- Engineering design.
- Forecasting.
- Biological conservation.
- Medical decisions.
- Homeland security.
- Resource economics.
- Monetary policy.
- Financial economics.
- Ellsberg and Allais paradoxes.
- Foraging by animals.
- Quantum indeterminacy.